MATH 551 - Problem Set 3

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1. By Menelaus' theorem we know that when a triangle is cut by a line, the product of the ratio of divided sides is -1. So, we know that $\frac{CB_1}{B_1A} \times \frac{AC_1}{C_1B} \times \frac{BA_1}{A_1C} = -1$. Well, we are given the majority of these lengths, so we know that $\frac{CB_1}{B_1A} \times \frac{1}{3} \times \frac{1}{4} = -1$, and we know the length AC to be 6, so we may write CB_1 as $6 + x$ where $AB_1 = x$. Thus we have $-\frac{6+x}{x} \times \frac{1}{3} \times \frac{1}{4} = -1 \Rightarrow -\frac{6+x}{12x} = -1 \Rightarrow 6 + x = 12x \Rightarrow x = 6/11$. So we have that $AB_1 = 6/11$.

2. We will use Desargues twice to prove this statement. We begin by observing $\triangle AMD$ and $\triangle ELB$. We know that the extended sides DM and BL intersect at point C (given), likewise AM and EL intersect at point F (given), and finally we know that AD and BE intersect at some point (given), call that intersection V. Well, because CF, AD, BE are concurrent (given), we know that our point V, C and F are on the same line. This means that $\triangle AMD$ and $\triangle ELB$ are perspective from a line (specifically, the VCF line). By Desargues, we know that these triangles thus must be perspective from a point (due to the bi-implication of Desargues). This point of perspective is O, as the a and b lines meet at O and go through A and E, D and B respectively. Thus our last vertices also must connect with O, so we have M and L on the same line as O.

We have shown that M and L are on the same line as O, so all that is left is to show that K is on this same line. We proceed similarly, now observing $\triangle AKD$ and $\triangle CLF$. We see that AK and CL intersect at B (given), and DK and FL intersect at E (given), and lastly AD and FC intersect at some point (given), which we've called V. Well, because we know V to be on the EB line (given), we have found that $\triangle AKD$ and $\triangle CLF$ are perspective from a line (the VEB line). Again, by Desargues (double implication), we have that $\triangle AKD$ and $\triangle CLF$ must be perspective with respect to a point, and that point of perspective is O as the a and b lines meet at O and go through A and C, D and F respectively. Thus our last vertices also must connect with O, so we have that K and L are on the same line as O.

We have shown that M and L are on the same line as O, and that K and L are on the same line as O, which means that O, K, L, M are all on the same line. $\square$